# UNIT - 2

## Problems

1) A resistive load of 10Ω is connected through a half-wave SCR circuit to 220V, 50HZ, single phase source. calculate the power delivered to the load for a firing angle of 60°. Find also the value of input power factor.

Soln

$V_s = 220V, \quad f = 50HZ, \quad R = 10\Omega$

firing angle $\alpha = 60°$, $V_m = \sqrt{2} \times V_s = \sqrt{2} \times 220 = 311.12V$

power delivered to the load = $V_{orms} \cdot I_{orms}$

$$V_{orms} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \frac{311.12}{2}\left[\frac{1}{\pi}\left(\pi - 60 \times \frac{\pi}{180} + \frac{\sin 2 \times 60}{2}\right)\right]^{1/2}$$

$$V_{orms} = 139.38V$$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{139.38}{10} = 13.938A$$

$$P_{ac} = 139.38 \times 13.938 = 1943W$$

$$\text{Input power factor} = \frac{V_{orms}}{V_s} = \frac{139.38}{220}$$

$$PF = 0.633 \text{ (lag)}.$$

2) A 1$\phi$ half wave converter is operated from a 120V, 60Hz supply. If the load is resistive of value 10 Ω delay angle is $\alpha = \frac{\pi}{3}$. Determine i) the efficiency ii) Form factor iii) ripple factor iv) transformer utilization factor v) peak inverse voltage of thyristor:

Soln

$V_s = 120V$, $R = 10\Omega$, $\alpha = \pi/3$.

$$V_{dc} = \frac{V_m}{2\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times 120}{2\pi}(1+\cos\pi/3)$$

$$= 40.5V$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{40.5}{10} = 4.05A$$

$$V_{orms} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right)\right]^{1/2} = \frac{\sqrt{2}\times 120}{2}\left[\frac{1}{\pi}\left(\pi-\frac{\pi}{3}+\frac{\sin 2\times 60}{2}\right)\right]^{1/2}$$

$$= 76.11 V$$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{76.11}{10} = 7.611 A$$

$$P_{dc} = V_{dc} I_{dc} = 40.5 \times 4.05 = 164.025 W$$

$$P_{ac} = V_{orms} I_{orms} = 76.11 \times 7.611 = 579.2 W$$

i) Rectification efficiency ($\eta$)

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{164.025}{579.2} \times 100 = 28.32\%$$

ii) Form factor (FF)

$$FF = \frac{\text{RMS value}}{\text{Average value}} = \frac{76.11}{40.5} = 1.88$$

iii) Ripple factor

$$RF = \sqrt{(FF)^2 - 1} = \sqrt{(1.88)^2 - 1} = 1.59.$$

iv) Transformer utilization factor (TUF)

$(I_s = I_{orms})$

$$TUF = \frac{P_{dc}}{V_s I_s} = \frac{164.025}{120 \times 7.611} = 0.1781$$

v) peak inverse voltage, PIV

$$PIV = V_m = \sqrt{2} \times V_s = \sqrt{2} \times 120 = 169.7V$$

3) A 1 $\phi$ ~~ph~~ half wave controlled rectifier has a purely resistive load R and delay angle is $\alpha = \pi/2$. Find i) the rectification efficiency (ii) Form factor, (iii) ripple factor (iv) transformer utilization factor (v) peak inverse voltage of the thyristor :

Soln

At $\alpha = \pi/2$

$$V_{dc} = \frac{V_m}{2\pi}(1 + \cos\alpha)$$

$$= \frac{V_m}{2\pi}(1 + \cos \pi/2) = 0.1592\ V_m$$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{0.1592\, V_m}{R}$$

$$V_{orms} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi - \pi/2 + \frac{\sin 2\,\pi/2}{2}\right)\right]^{1/2}$$

$$= 0.3536\, V_m$$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{0.3536\, V_m}{R}$$

$$P_{dc} = V_{dc}\, I_{dc} = (0.1592\, V_m)\left(\frac{0.1592\, V_m}{R}\right)$$

$$= \frac{(0.1592\, V_m)^2}{R}$$

$$P_{ac} = V_{orms}\, I_{orms} = \frac{(0.3536\, V_m).(0.3536\, V_m)}{R}.$$

$$= \frac{(0.3536\, V_m)^2}{R}.$$

i) Rectification efficiency :

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{(0.1592\, V_m)^2/R}{(0.3536\, V_m)^2/R} = 20.27\%$$

ii) Form factor (FF):

$$FF = \frac{V_{orms}}{V_{dc}} = \frac{0.3536\, V_m}{0.1592\, V_m} = 2.221$$

iii) Ripple factor RF

$$RF = \sqrt{(FF)^2 - 1} = \sqrt{(2.221)^2 - 1} = 1.983.$$

iv) TUF

$$TUF = \frac{P_{dc}}{V_s I_s} = \frac{(0.1592\, V_m)^2/R}{\frac{V_m}{\sqrt{2}} \times \frac{0.3536\, V_m}{R}} = 0.1014.$$

v) $PIV = V_m$

A) Design a half wave controlled rectifier circuit to produce an average voltage of 40V across a 100Ω load resistor from a 120V rms, 60Hz ac source. Determine the power absorbed by the resistance and the power factor.

Soln

$V_{dc} = 40V$, $R = 100\Omega$, $V_s = 120V$, $V_m = \sqrt{2} \times V_s = \sqrt{2} \times 120 = 169.7V$

$$V_{dc} = \frac{V_m}{2\pi}(1+\cos\alpha)$$

$$\alpha = \cos^{-1}\left[V_{dc}\left(\frac{2\pi}{V_m}\right) - 1\right] = \cos^{-1}\left[40\left(\frac{2\pi}{\sqrt{2}\times 120}\right) - 1\right] = 61.2^\circ$$

$$V_{orms} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \frac{169.7}{2}\left[\frac{1}{\pi}\left(\pi - \frac{61.2 \times \pi}{180} + \frac{\sin 2 \times 61.2}{2}\right)\right]^{1/2}$$

$$= 75.6V$$

Load power

$$P_{ac} = \frac{V_{orms}^2}{R} = \frac{(75.6)^2}{100} = 57.1\ W$$

The power factor of the circuit is

$$PF = \frac{V_{orms}}{V_s} = \frac{75.6}{120} = 0.63\ (lag).$$

5) A $1\phi$ half wave rectifier is used to supply power to a load of impedance $10\Omega$ from 230V, 50Hz, AC supply at the firing angle of 30°. Calculate the load current.

Soln

$V_S = 230V$, $R = 10\Omega$, $\alpha = 30°$, $V_m = \sqrt{2}\, V_S = \sqrt{2} \times 230$.

Average o/p voltage $V_{dc} = \frac{V_m}{2\pi}(1+\cos\alpha)$

$$= \frac{\sqrt{2}\times 230}{2\pi}(1+\cos 30°)$$

$$\boxed{V_{dc} = 96.6V}$$

load current $I_{dc} = \frac{V_{dc}}{R} = \frac{96.6}{10} = 9.66A$

$$\boxed{I_{dc} = 9.66A}$$

6) A 220V, 50Hz, $1\phi$ ac voltage source supplies a resistive load of $20\Omega$ through a half wave controlled rectifier circuit. If the thyristor delay angle is fixed at 90 degrees, determine a) ratio of rectification (b) form factor (c) ripple factor d) TUF (e) peak inverse voltage. Assume that the turns ratio of transformer supplying the rectifier circuit is unity

Soln

$V_S = 220V$, $R = 20\Omega$, $\alpha = 90°$.

$V_{dc} = \frac{V_m}{2\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times 220}{2\pi}(1+\cos 90°)$

$= 49.52V$

$$I_{dc} = \frac{V_{dc}}{R} = \frac{49.5}{20} = 2.475A$$

$$V_{orms} = \frac{V_m}{2}\left[\frac{1}{\pi}\left(\pi - \alpha + \frac{\sin 2\alpha}{2}\right)\right]^{1/2}$$

$$= \frac{\sqrt{2}\times 220}{2}\left[\frac{1}{\pi}\left(\pi - \frac{\pi}{2} + \frac{\sin 2\times 90}{2}\right)\right]^{1/2}$$

$$= 110V.$$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{110}{20} = 5.5A$$

$$P_{dc} = V_{dc} I_{dc} = 49.5 \times 2.475 = 122.5W$$

$$P_{ac} = V_{orms} I_{orms} = 110 \times 5.5 = 605 W$$

a) Ratio of rectification :

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{122.5}{605} \times 100 = 20.24\%$$

b) Form factor (FF)

$$FF = \frac{\text{RMS value}}{\text{Average value}} = \frac{110}{49.5} = 2.22$$

c) voltage ripple factor

$$RF = \sqrt{(FF)^2 - 1} = \sqrt{(2.2)^2 - 1} = 1.982$$

d) TUF

$$I_s = I_{orms}$$

$$TUF = \frac{P_{dc}}{V_s I_s} = \frac{122.5}{220 \times 5.5} = 0.101.$$

e) peak inverse voltage:

$$PIV = V_m = \sqrt{2} \times V_s = \sqrt{2} \times 220 = 311.12V$$

8) A 1ϕ half wave rectifier with a freewheeling is used to supply a heavily inductive load upto 15A from a 240V ac supply. Determine the mean load voltage for firing angles of 0°, 30°, 60°, 120° and 180°, neglecting the thyristor voltage drops. Specify the required rating of the thyristor and diode.

Soln

$$V_{dc} = \frac{\sqrt{2}V_s}{2\pi}(1+\cos\alpha)$$

For $\alpha = 0°$

$$V_{dc} = \frac{\sqrt{2}\times 240}{2\pi}(1+\cos 0°) = 108V.$$

For $\alpha = 30°$

$$V_{dc} = \frac{\sqrt{2}\times 240}{2\pi}(1+\cos 30°) = 100.8 V$$

For $\alpha = 60°$

$$V_{dc} = \frac{\sqrt{2}\times 240}{2\pi}(1+\cos 60°) = 81.02V$$

For $\alpha = 120°$

$$V_{dc} = \frac{\sqrt{2}\times 240}{2\pi}(1+\cos 120°) = 27V$$

For $\alpha = 180°$

$$V_{dc} = \frac{\sqrt{2}\times 240}{2\pi}(1+\cos 180°) = 0V$$

Thyristor Rating:

peak inverse voltage $V_m = \sqrt{2}\times V_s$

$$= \sqrt{2}\times 240 = 340V.$$

The thyristor will conduct for a maximum duration when delay angle $\alpha = 0°$ of one cycle. Using two equal time intervals the rms current rating can be calculated as

$$I_{Trms} = \left[\frac{15^2 + 0^2}{2}\right] = 10.6A$$

Diode Rating:

Peak inverse voltage $V_m = \sqrt{2} \times V_s = 340\,V$.

As the delay angle approaches 180°, the diode will conduct for almost the whole cycle, hence the required current rating would be 15A.

9) 1 ϕ Full wave converter is operated from a 120V, 60HZ for a resistive load of 10Ω. If the average output voltage is 25% of maximum possible average o/p voltage, find i) the delay angle, ii) average and rms output currents iii) average and rms thyristor currents.

Soln

$V_s = 120\,V,\ f = 60\,HZ\ \ R = 10\,\Omega,\ V_m = \sqrt{2} \times V_s = \sqrt{2} \times 120 = 169.7V$

For single phase full converter with R load.

$$V_{dc} = \frac{V_m}{\pi}(1 + \cos\alpha)$$

at $\alpha = 0°$, we can get maximum possible average o/p voltage

$$V_{dc,max} = \frac{2V_m}{\pi} = \frac{2 \times \sqrt{2} \times 120}{\pi} = 108.03V$$

the average o/p voltage is 25% of $V_{dc\,max}$

$$V_{dc} = 0.25 \times 108.03 = 27V.$$

i) Delay angle ($\alpha$):

$$V_{dc} = \frac{V_m}{\pi}(1 + \cos\alpha).$$

$$\alpha = \cos^{-1}\left[\frac{\pi \times V_{dc}}{V_m} - 1\right] = \cos^{-1}\left[\frac{\pi \times 27}{\sqrt{2} \times 120} - 1\right]$$

$$\boxed{\alpha = 120°}$$

~~Bio~~ ii) Average o/p current :

$$I_{dc} = \frac{V_{dc}}{R} = \frac{27}{10} = 2.7A$$

$$V_{orms} = V_m\left[\frac{\pi-\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right]^{1/2} = ~~53V~~$$

$$= \sqrt{2}\times120\left[\frac{\pi-2\pi/3}{2\pi} + \frac{\sin 2\times2\pi/3}{4\pi}\right]^{1/2}$$

$$= 53V.$$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{53}{10} = 5.3A.$$

iii) Average thyristor current :

$$I_{TA} = \frac{I_{dc}}{2} = \frac{2.7}{2} = 1.35A$$

RMS thyristor current

$$I_{TR} = \frac{I_{orms}}{\sqrt{2}} = \frac{5.3}{\sqrt{2}} = 3.719A.$$

(10) A Full wave controlled bridge rectifier has an ac i/p of 160V rms at 50Hz and a 10Ω load resistor. The delay angle is 45°. Determine the average current in the load, the power absorbed by the load and the input power factor.

Soln

$V_s = 160V$, $f = 50Hz$, $R = 10\Omega$, $\alpha = 45°$

i) Average load voltage :

$$V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times160}{\pi}(1+\cos 45°) = 122.95V.$$

ii) Average load current :

$$I_{dc} = \frac{V_{dc}}{R} = \frac{122.95}{10} = 12.295A.$$

ii) RMS load voltage $V_{orms} = V_m \left[ \frac{\pi - \alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi} \right]^{1/2}$

$$= \sqrt{2} \times 160 \left[ \frac{\pi - \left(45 \times \frac{\pi}{180}\right)}{2\pi} + \frac{\sin 2 \times 45}{4\pi} \right]^{1/2} = 152.55V$$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{152.55}{10} = 15.255A.$$

power absorbed by the load.

$$= I_{orms}^2 R = (15.255)^2 \times 10 = 2327.15\ W$$

iii) input power factor.

$$= \frac{V_{orms}}{V_s} = \frac{152.55}{160} = 0.9534\ (lag).$$

11) A $1\phi$ full converter bridge is connected to RLE load. The source voltage is 230V, 50HZ. The average load current of 10A continuous over the working range. For $R = 0.4\Omega$ and $L = 2mH$ compute a) Firing angle for $E = 120V$ b) firing angle for $E = -120V$.

Soln

$$V_s = 230V, \quad I_{dc} = 10A, \quad R = 0.4\Omega, \quad L = 2mH$$

a) For $E = 120V$, the full converter is operating as a controlled rectifier (i.e. rectification mode).

$$\frac{2V_m}{\pi} \cos\alpha = E + I_{dc} R.$$

$$\frac{2 \times \sqrt{2} \times 230}{\pi} \cos\alpha = 120 + 10 \times 0.4 = 124V.$$

$$\alpha = 53.21^\circ.$$

For $\alpha = 53.21^\circ$, power flows from ac source to dc load.

b) For $E = -120V$, the full converter operating as a line commutated inverter.

$$\frac{2 \times \sqrt{2} \times 230}{\pi} \cos\alpha = -120 + 10 \times 0.4 = -116V.$$

$$\alpha = 124.1^\circ$$

For $\alpha = 124.1^\circ$ the power flows from dc source to ac load.

12) A 1φ full converter is supplied from 220V, 50Hz source. The load consist of $R = 12\Omega$ and a large inductance so as to render the load current constant. For a firing angle delay of 45°, determine i) average o/p voltage ii) average o/p current iii) average and rms values of thyristor currents and iv) power factor.

Soln

i) For a single phase full converter average o/p voltage $V_0$

$$V_{dc} = \frac{2V_m}{\pi} \cos\alpha = \frac{2 \times \sqrt{2} \times 220}{\pi} \cos 45^\circ = 140.05\,V$$

ii) Average o/p current

$$I_{dc} = \frac{V_{dc}}{R} = \frac{140.05}{12} = 11.67A.$$

iii) average value of thyristor current

$$I_{TA} = \frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} I_{dc}\, d(\omega t)$$

$$= \frac{1}{2\pi} \left[\pi - \alpha + \alpha\right] = \frac{I_{dc}}{2} = \frac{11.67}{2} = 5.833A$$

RMS value of thyristor current

$$I_{TR} = \left[\frac{1}{2\pi}\int_{\alpha}^{\pi+\alpha} I_{dc}^2 \, d(\omega t)\right]^{1/2}$$

$$= \left[\frac{I_{dc}^2}{2\pi}[\pi+\alpha-\alpha]\right]^{1/2} = \frac{I_{dc}}{\sqrt{2}} = \frac{11.67}{\sqrt{2}} = 8.25A.$$

iv) RMS value of source current $I_s$.

$$I_s = \left[\frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} I_{dc}^2 \, d(\omega t)\right]^{1/2} = I_{dc} = 11.67A$$

load power = $V_{dc} I_{dc} = 140.05 \times 11.67$

$= 1634.38 W.$

input power = $V_s I_s \cos\phi_s = 220 \times 11.67 \times \cos\phi_s$

For no loss in the power converter

$V_s I_s \cos\phi_s = V_o I_o \Rightarrow \cos\phi_s = \frac{V_o I_o}{V_s I_s}$

$$\cos\phi_s = \frac{1634.38}{220 \times 11.67}$$

$\cos\phi_s = 0.6365$ (lag).

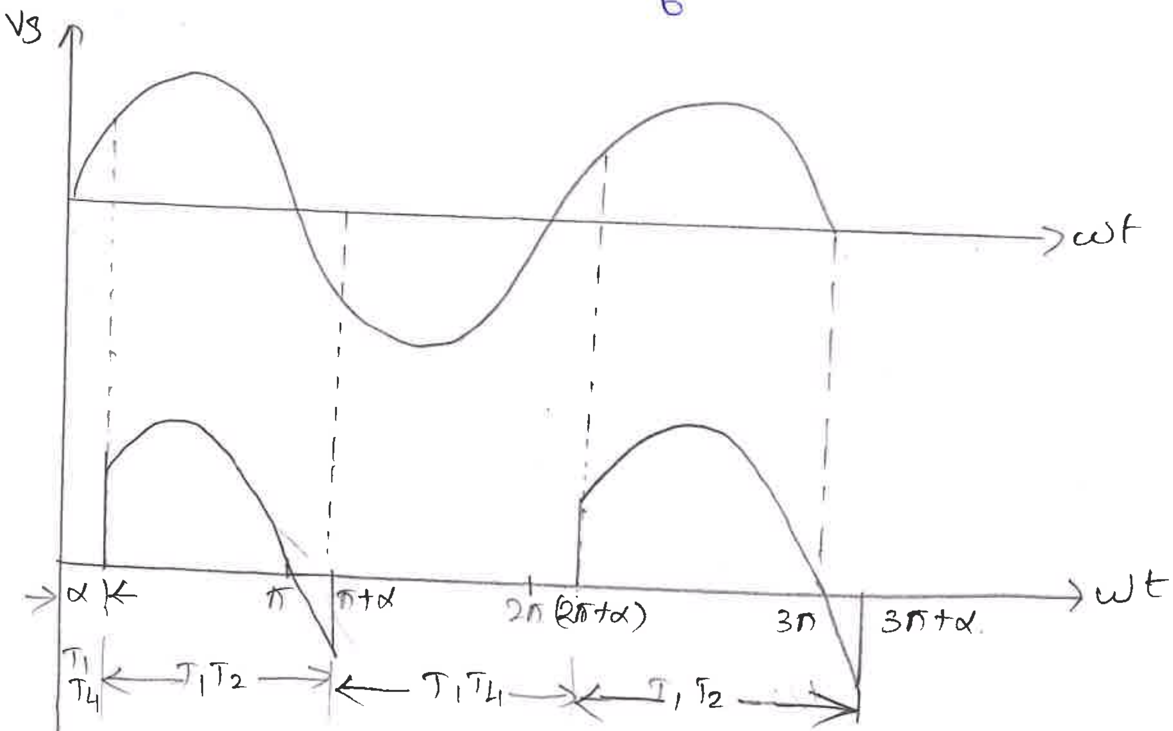
13) A 1φ ~~half~~ full converter feeds power to RLE load with $R = 6\Omega$ $L = 6\,mH$, and $E = 60V$. The ac source voltage is 230v, 50Hz, For continuous conduction find the average value of load current for a firing angle of 50°. In case one of the four SCRs gets open circuited due to a fault, find the new value of average load current taking the o/p current as continuous. Sketch wave form for the new o/p voltage & indicate the conduction of

various SCRs.

Soln

$V_{dc} = \frac{2V_m}{\pi}\cos\alpha = \frac{2\times\sqrt{2}\times 230}{\pi}\cos 50^\circ = 133.08\,V$

$I_{dc} = \frac{V_{dc}-E}{R} = \frac{133.08-60}{6} = 12.181\,A.$




Suppose SCR $T_3$ is damaged and is open circuited with this o/p voltage waveform $V_0$ is shown figure. Initially suppose $T_1, T_2$ are conducting from α to π

At $\omega t = \pi+\alpha$ when $T_3, T_4$ are triggered. only $T_4$ is turned on and as a result, load current free wheels through $T_1/T_4$ and is zero until $T_1, T_2$ are triggered again at $\omega t = 2\pi+\alpha$.

average o/p voltage $\cdot V_{dc} = \frac{1}{2\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin\omega t\, d(\omega t)$

$= \frac{V_m}{\pi}\cos\alpha.$

$= \frac{\sqrt{2}\times 230}{\pi}\cos 50^\circ = 66.542\,V$

$$I_{dc} = \frac{V_0 - E}{R} = \frac{66.542 - 60}{6} = 1.0903A.$$

14) A $1\phi$ full converter delivers power to RLE load with $R = 0.5\Omega$, $L = 6.5mH$ and $E = 10V$. The ac supply voltage is 230V, 50HZ. For the continuous conduction, determine the average o/p voltage and load current at delay angle $\alpha = 60°$.

Soln

$R = 0.5\Omega$, $L = 6.5mH$, $E = 10V$, $V_s = 230V$, $\alpha = 60°$

Average o/p voltage $V_{dc} = \frac{2V_m}{\pi} \cos\alpha = \frac{2 \times \sqrt{2} \times 230}{\pi} \cos 60$

$= 103.53V.$

Average load current $I_{dc}$ :-

$V_{dc} = E + I_{dc} R.$

$$I_{dc} = \frac{V_{dc} - E}{R} = \frac{103.53 - 10}{0.5} = 187.06A.$$

15) A $1\phi$ full converter is supplied from 230V, 50HZ source. The load consist of $R = 10\Omega$ and a large inductance so as to render the load current constant. For a firing angle delay of $30°$. determine

1. Average o/p voltage
2. Average o/p current
3. Average & rms value of thyristor currents
4. The power factor.

Soln

$V_s = 230V$, $R = 10\Omega$, $\alpha = 30°$ . $V_m = \sqrt{2} \times V_s = \sqrt{2} \times 230$

$= 325.27V$.

i) Average o/p voltage

$$V_{dc} = \frac{2V_m}{\pi} \cos\alpha = \frac{2 \times 325.27}{\pi} \cos 30° = 179.33V$$

ii) average o/p current $I_{dc} = \frac{V_{dc}}{R}$

$$= \frac{179.33}{10} = 17.933A.$$

iii) Average value of thyristor current ($I_{TA}$)

$$I_{TA} = \frac{I_{dc}}{2} = \frac{17.933}{2} = 8.966A.$$

RMS value of thyristor current

$$I_{TR} = \frac{I_{dc}}{\sqrt{2}} = \frac{17.933}{\sqrt{2}} = 12.68A$$

iv) input power factor ($\cos\phi$)

load power $= V_{dc} I_{dc} = 179.33 \times 17.933$

$= 3215.9$ W

i/p power $= V_s I_s \cos\phi_s$

$= 230 \times 17.933 \cos\phi_s$.

For no loss in the power converter

I/P power = o/p power.

$$V_s I_s \cos\phi_s = V_{dc} I_{dc}$$

$$\cos\phi_s = \frac{V_{dc} I_{dc}}{V_s I_s} = \frac{3215.9}{230 \times 17.933} = 0.779 \text{ (lag)}$$

16) The half controlled 1$\phi$ bridge circuit is supplied at 120V. Neglecting volt drops, determine the mean load voltage at firing delay angles of 0°, 90° & 180°. If the load is highly inductive taking 20A, determine the required device ratings.

Soln

$V_s = 120V, \ \alpha = 0°, 90°, 180°, \ I_{dc} = 20A.$

$$V_{dc} = \frac{V_m}{\pi}(1 + \cos\alpha)$$

For $\alpha = 0°$

$$V_{dc} = \frac{\sqrt{2} \times 120}{\pi}(1 + \cos 0°) = 108V.$$

For $\alpha = 90°$; $V_{dc} = \frac{\sqrt{2} \times 120}{\pi}(1 + \cos 90°) = 54V.$

For $\alpha = 180°$; $V_{dc} = \frac{\sqrt{2} \times 120}{\pi}(1 + \cos 180°) = 0V.$

Each thyristor and diode must withstand $V_m = \sqrt{2}V_s = \sqrt{2} \times 120 = 170V$. The bridge components conduct for a maximum of one half cycle, hence for level current

$$I_{rms} = \frac{20}{\sqrt{2}} = 14.16A.$$

The freewheeling diode will conduct for almost the complete cycle when 180°, therefore it must be rated to 20A.

17) A 1$\phi$ semiconverter is operated from a 120 V, 6HZ supply and load is resistive of R = 10 ohm. If the average o/p voltage is 25% of the maximum possible average o/p voltage, calculate i) delay angle ii) rms and average o/p currents iii) average & rms thyristor currents iv) i/p power factor.

Soln

$V_s = 120V$, $f = 60 HZ$, $V_m = \sqrt{2} \times 120 = 169.7 V$ $R = 10\Omega$

i) Delay angle

$$V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha)$$

$$V_{dc\,max} = \frac{2V_m}{\pi} = \frac{2\times\sqrt{2}\times 120}{\pi} = 108.03V.$$

The average o/p voltage is 25% of maximum possible o/p voltage

$$V_{dc} = 0.25 \times 108.03 = 27V.$$

$$V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha).$$

$$\alpha = \cos^{-1}\left[\frac{V_{dc} \times \pi}{V_m} - 1\right]$$

$$\alpha = \cos^{-1}\left[\frac{27\times\pi}{169.7} - 1\right] = 120^\circ.$$

ii) average and RMS o/p current

$$I_{dc} = \frac{V_{dc}}{R} = \frac{27}{10} = 2.7A.$$

$$\boxed{I_{dc} = 2.7A}$$

$$V_{orms} = V_m\left[\frac{\pi-\alpha}{2\pi} + \frac{\sin 2\alpha}{4\pi}\right]^{1/2}$$

$$= 169.7 \left[ \frac{\pi - 120 \times \frac{\pi}{180}}{2\pi} + \frac{\sin 2 \times 120}{4\pi} \right]^{1/2}$$

$V_{orms} = 53.04\ V.$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{53.04}{10} = 5.304\ A.$$

iii) average thyristor current

$$I_{TA} = \frac{I_{dc}}{2} = \frac{2.7}{2} = 1.35A$$

RMS thyristor current

$$I_{TR} = \frac{I_{orms}}{\sqrt{2}} = \frac{5.304}{\sqrt{2}} = 3.75A.$$

iv) Input power factor:

$$\cos\phi = \frac{\text{power delivered to load}}{\text{input VA}} = \frac{V_{orms} I_{orms}}{V_s I_s}.$$

$$= \frac{V_{orms}}{V_s} = \frac{53.04}{120} = 0.442\ (lag).$$

18) what is the o/p voltage for the triggering angle $\alpha = 90°$ in a $1\phi$ 230v half controlled converter?

Soln

Triggering angle $\alpha = 90°$. $V_s = 230V$.

$$o/p\ V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha) = \frac{\sqrt{2} \times 230}{\pi}(1+\cos 90°)$$

$= 103.536\ V.$

19) A single phase half controlled rectifier supplies a load of R=10Ω and L=10mH. It is operated from 230V, 50Hz, ac mains. calculate Vdc, Idc when α = 60°.

Soln

$R = 10\Omega$, $L = 10\,mH$, $V_s = 230V$, $\alpha = 60^\circ$

Average o/p voltage $V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times 230}{\pi}(1+\cos 60)$

$$V_{dc} = 155.30V$$

Average load current $I_{dc} = \frac{V_{dc}}{R} = \frac{155.3}{10} = 15.53A.$

20) A 1φ half controlled converter is connected to a resistance R=2Ω, inductance L=0.3H and emf E=50V load. For a trigger angle of 20°. If the load current can be assumed to be of constant value estimate the average load current and the o/p factor. The supply voltage is 230V, 50Hz.

Soln

$R = 2\Omega$, $L = 0.3H$, $E = 50V$, $\alpha = 20^\circ$, $V_s = 230V$,

$V_m = \sqrt{2}\times V_s = \sqrt{2}\times 230 = 325.26V$

Average o/p voltage $V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha) = \frac{325.26}{\pi}(1+\cos 20^\circ)$

$$V_{dc} = 200.8V$$

Average load current $I_{dc}$ :-

$V_{dc} = E + I_{dc}R$

$I_{dc} = \frac{V_{dc} - E}{R} = \frac{200.8-50}{2} = 75.4A$

$$I_{dc} = 75.4A$$

Input power factor :

Source current $I_s = I_{dc}\left[\frac{\pi-\alpha}{\pi}\right]^{1/2}$

$$= 75.4\left[\frac{\pi - 20\times\pi/180}{\pi}\right]^{1/2} = 71.08A$$

Input power factor $\cos\phi = \frac{V_{dc}I_{dc}}{V_sI_s} = \frac{200.8\times75.4}{230\times71.08}$

$$\boxed{\cos\phi = 0.926\,(\text{lag})}$$

21) A single phase bridge converter with a free wheeling diode feeds an RL load. The load resistance is 7.5 Ω and inductance is very large providing ripple free load current. The converter is supplied by 120V, 1φ supply at a frequency of 50 HZ. Determine the average load voltage, load current and power factor at a firing angle of 60°.

Soln

$R = 7.5\Omega$, $V_s = 120V$, $\alpha = 60°$

Average load voltage $V_{dc} = \frac{V_m}{\pi}(1+\cos\alpha) = \frac{\sqrt{2}\times120}{\pi}(1+\cos60°)$

$$\boxed{V_{dc} = 81.03V}$$

Average load current $I_{dc} = \frac{V_{dc}}{R} = \frac{81.03}{7.5} = 10.804A$

RMS value of source current $I_s = I_{dc}\left[\frac{\pi-\alpha}{\pi}\right]^{1/2}$

$$= 10.804\left[\frac{\pi - \frac{\pi}{180}\times60}{\pi}\right]^{1/2}$$

$$= 8.821A$$

$$\text{Input power factor} = \frac{V_{dc} I_{dc}}{V_s I_s} = \frac{81.03 \times 10.804}{120 \times 8.821}$$

$$\boxed{\cos\phi = 0.827 \text{ (lag)}}$$

22) A $1\phi$ fully controlled bridge rectifier supplies an inductive load. assuming that the o/p current is virtually constant, and is equal to $I_{dc}$, determine the following performance measure, if the supply voltage is 230V and if the firing angle 30°.

i) Average o/p voltage ii) Supply RMS current
iii) Supply fundamental RMS current iv) Fundamental power factor
v) supply power factor vi) supply harmonic factor
vii) voltage ripple factor.

Soln

i) Average o/p voltage:

$$V_{dc} = \frac{2V_m}{\pi}\cos\alpha = \frac{2\times\sqrt{2}\times 230}{\pi}\cos 30° = 179.33V$$

ii) Supply RMS current $= I_{dc}$

iii) Supply fundamental RMS current $= \frac{2\sqrt{2}}{\pi} I_{dc}$

iv) Fundamental power factor or displacement factor

$$= \cos\alpha = \cos 30 = 0.866$$

v) Supply power factor:

$$= \frac{2\sqrt{2}}{\pi}\cos\alpha = \frac{2\sqrt{2}}{\pi}\cos 30° = 0.78 \text{ (lag)}$$

vi) supply harmonic factor $= \sqrt{\frac{\pi^2}{8} - 1} = 0.483$

vii) voltage ripple factor $= \left(\frac{\pi^2}{8\cos^2\alpha} - 1\right)^{1/2} = \left(\frac{\pi^2}{8\cos 30} - 1\right)^{1/2} = 0.803.$

23) A 3ϕ half wave controlled rectifier has a supply of 200V/ph. Determine the load voltage for delay angle of 0°, 30°, 60°. assuming a thyristor voltage drop of 1.5V and continuous load current.

Soln

$$V_{dc} = \frac{3\sqrt{3}\, V_m}{2\pi} \cos\alpha - \text{Voltage drop}$$

For $\alpha = 0°$ :

$(V_m = \sqrt{2}\, V_s)$

$$V_{dc} = \frac{3\sqrt{3}\cdot\sqrt{2} \times 200}{2\pi} \cos 0° - 1.5 = 232.4V.$$

For $\alpha = 30°$

$$V_{dc} = \frac{3\sqrt{3}\sqrt{2} \times 200}{2\pi} \cos 30° - 1.5 = 201.07V$$

For $\alpha = 60°$.

$$V_{dc} = \frac{3\sqrt{3}\,\sqrt{2} \times 200}{2\pi} \cos 60° - 1.5 = 115.45V$$

24) A 3ϕ half wave controlled rectifier is connected to a 380V (line) supply. The load current is constant at 32A, and is independent of firing angle. Find the average load voltage at firing angle of 0° and 45°; given that the thyristor have a forward voltage drop of 1.2V. what value of current and peak reverse voltage rating will the thyristor require and what will be the average power dissipation in each thyristor?

Soln

i) For $\alpha = 0^\circ$

$$V_{dc} = \frac{3\sqrt{3}V_m}{2\pi}\cos 3\alpha - V_t$$

$$= \frac{3\sqrt{3} \times 380 \times \sqrt{2}}{2\pi \times \sqrt{3}}\cos 0^\circ - 1.2 = 255.4V.$$

ii) For $\alpha = 45^\circ$

$$V_{dc} = \frac{3\sqrt{3} \times 380 \times \sqrt{2}}{2\pi \times \sqrt{3}}\cos 45^\circ - 1.2 = 180.2V$$

**Ratings:**

$$I_{rms} = \frac{32}{\sqrt{3}} = 18.47A.$$

For $3\phi$ half controlled rectifiers, the reverse voltage can be seen to be the difference b/w the two phase voltage ie the line voltage of the three phase supply.

$$PIV = \sqrt{2}\, V_{line}$$

$$= \sqrt{2} \times \sqrt{3} \times V_{phase}$$

$$= \sqrt{2} \times 380 = 537.4V.$$

Now, the average power dissipated in the thyristor is obtained by averaging the instantaneous power dissipation over one cycle.

$$\therefore \text{Average power} = \frac{1}{2\pi}\int_{\alpha}^{\alpha+\frac{2\pi}{3}} V_{Th}\, I_{Th}\, d(\omega t).$$

$$= \frac{V_{Th} \cdot I_{Th}}{3}$$

$$= \frac{1.2 \times 32}{3} = 12.8\,W$$

25) The $3\phi$ half wave controlled rectifier is operated from a $3\phi$ wye connected 440V, 50HZ, supply and load resistance R is 20Ω. If the average o/p voltage is 50% of the maximum possible average voltage, calculate i) delay angle ii) rms and average o/p currents iii) average and rms thyristor currents iv) rectification efficiency v) TUF vi) i/p power factor

Soln

$V_L = 440V$, $f = 50HZ$, $R = 20\Omega$

$$V_m = \frac{\sqrt{2} \times V_L}{\sqrt{3}} = \frac{\sqrt{2} \times 440}{\sqrt{3}} = 359.2\,V$$

$$V_{dc} = \frac{3\sqrt{3}\,V_m}{2\pi} \cos\alpha$$

$\alpha = 0°$

$$V_{dc\,max} = \frac{3\sqrt{3}\,V_m}{2\pi} = \frac{3\sqrt{3} \times 359.2}{2\pi}$$

$V_{dc\,max} = 297.05\,V$

But average o/p voltage is 50% of the maximum possible average voltage.

$V_n = 0.50$

$V_{dc} = 0.50 \times 148.52 = 148.525\,V$

i) Delay angle:

$$V_n = \frac{1}{\sqrt{3}}\left[1+\cos\left(\alpha+\frac{\pi}{6}\right)\right]$$

$$0.5\times\sqrt{3} = 1+\cos\left(\alpha+\frac{\pi}{6}\right)$$

$$-0.1339 = \cos\left(\alpha+\frac{\pi}{6}\right)$$

$$97.69 = \alpha+\pi/6$$

$$\alpha = 97.69 - \pi/6$$

$$\boxed{\alpha = 67.69^\circ}$$

ii) Average load current:

$$I_{dc} = \frac{V_{dc}}{R} = \frac{148.525}{20} = 7.426\ A.$$

$$V_{orms} = \sqrt{3}\,V_m\left[\frac{5}{24} - \frac{\alpha}{4\pi} + \frac{1}{8\pi}\sin\left(\frac{\pi}{3}+2\alpha\right)\right]^{1/2}$$

$$= \sqrt{3}\times 359.2\left[\frac{5}{24} - \frac{67.69}{4\pi} + \frac{1}{8\pi}\sin\left(\frac{\pi}{3}+2\alpha\right)\right]^{1/2}$$

$$= 210.169\ V.$$

$$I_{orms} = \frac{V_{orms}}{R} = \frac{210.169}{20} = 10.5\ A.$$

iii) Average and rms thyristor current:

$$I_{AT} = \frac{I_{dc}}{3} = \frac{7.426}{3} = 2.475\ A$$

iv) Rectification efficiency :

$$\eta = \frac{P_{dc}}{P_{ac}} = \frac{V_{dc} I_{dc}}{V_{orms} I_{orms}} = \frac{148.525 \times 7.426}{210.169 \times 10.50}$$

$= 49.98\%$.

v) RMS input current is the same as the thyristor rms current.

Input volt ampere $VI = 3V_s I_s$

$= 3 \times 254.03 \times 6.062 = 4619.24$ VA.

$$TUF = \frac{P_{dc}}{3V_s I_s} \times 100$$

$$= \frac{148.525 \times 7.426}{4619.24} \times 100$$

$= 23.87\%$.

vi) Input power factor :

$$= \frac{P_o}{\text{Input VA}} = \frac{I_{orms}^2 R}{3 V_s I_s}$$

$$= \frac{(10.50)^2 \times 20}{4619.24} = 0.477 (\text{lag})$$

26) A 3$\phi$ full controlled rectifier is supplied from a 3$\phi$, 400V, 50Hz supply. The load current is continuous and has negligible ripple. If the average load current $I_{dc} = 150A$ and the commutating inductance $L_c = 0.1\,mH$, determine the overlap angle when $\alpha = 45^\circ$

Soln

$V_S = 400V$, $f = 50Hz$, $I_{dc} = 150A$, $L_c = L_S = 0.1\,mH$

$\alpha = 45^\circ$

$V_{mL} = \sqrt{2} \times 400 = 565.68V$.

For $\alpha = 45^\circ$

$$V_{dc} = \frac{3V_{mL}}{\pi}\cos\alpha - \frac{3\omega L_s}{\pi} I_{dc}$$

$$= \frac{3 \times 565.68}{\pi}\cos 45^\circ - \frac{3 \times 2\pi \times 50 \times 0.1 \times 10^{-3} \times 150}{\pi}$$

$$= 382 - 4.5$$

$$\boxed{V_{dc} = 377.5\ V}$$

$$V_{dc} = \frac{3V_{mL}}{\pi}\cos(\alpha+\mu) + \frac{3\omega L_s I_{dc}}{\pi}$$

$$= \frac{3 \times 565.68}{\pi}\cos(45^\circ+\mu) + \frac{3 \times 2\pi \times 50 \times 0.1 \times 10^{-3} \times 150}{\pi}$$

$377.5 = 540.184\cos(45^\circ+\mu) + 4.5$

$540.184\cos(45^\circ+\mu) = 373$

$\cos(45^\circ+\mu) = 0.6905$

$$45^\circ + \mu = \cos^{-1}(0.6905)$$

$$45^\circ + \mu = 46.83$$

$$\mu = 46.83 - 45$$

$$\boxed{\mu = 1.83^\circ}$$

27) A 3$\phi$ Fully controlled rectifier to a supply voltage 230V per phase and frequency 50HZ. The source inductance is 3mH. The load current on dc side is constant at 15A. If the load consist of a dc source voltage of 400V having an internal resistance of 1Ω, compute the following: i) Firing angle ii) overlap angle.

Soln

$V_s = 230V/ph$ $f = 50HZ$, $L_s = 3mH$, $I_{dc} = 15A$, $E = 400V$

$R_i = 1\Omega$

$$V_{mL} = \sqrt{2} \times \sqrt{3} \times 230 = 563.38\,V$$

i) Firing angle:

$$V_{dc} = E + I_{dc}R = 400 + 15 \times 1 = 415\,V.$$

$$V_{dc} = \frac{3V_{mL}}{\pi}\cos\alpha - \frac{3\omega L_s}{\pi} I_{dc}.$$

$$415 = \frac{3 \times 563.38}{\pi}\cos\alpha - \frac{3 \times 2\pi \times 50 \times 3 \times 10^{-3}}{\pi} \times 15$$

$$\alpha = 37.20^\circ$$

ii) over lap angle :

$$V_{dc} = \frac{3V_{mL}}{\pi}\cos(\alpha+\mu) + \frac{3\omega L_s I_{dc}}{\pi}$$

$$415 = \frac{3\times 563.38}{\pi}\cos(37.2+\mu) + \frac{3\times 2\pi\times 50\times 3\times 10^{-3}\times 15}{\pi}$$

$$\mu = 4.53^\circ.$$

28) A $3\phi$ fully controlled rectifier is connected to $3\phi$ ac supply of 400V, 50Hz and operates with a firing angle $\alpha = 60^\circ$. The load current is maintained constant at 25A and the load voltage is 250V. compute i) Source inductance $L_s$, ii) Load resistance R, iii) over lap angle $\mu$)

Soln

$V_L = 400\,V$, $f = 50\,Hz$, $\alpha = 60^\circ$, $I_{dc} = 25A$, $V_{dc} = 250\,V$

i) Source inductance $L_s$ :

$$V_{mL} = 400\sqrt{2} = 565.68\,V$$

$$V_{dc} = \frac{3V_{mL}}{\pi}\cos\alpha - \frac{3\omega L_s}{\pi} I_{dc}$$

$$250 = \frac{3\times 565.68}{\pi}\cos 60^\circ - \frac{3\times 2\pi\times 50\times L_s\times 25}{\pi}$$

$$250 = 270.09 - 7500\,L_s$$

$$7500 L_s = 20.09$$

$$\boxed{L_s = 2.6\,mH}$$

ii) load resistance :

$$R = \frac{V_{dc}}{I_{dc}} = \frac{250}{25} = 10\Omega$$

$$\boxed{R = 10\Omega}$$

iii) over lap angle :

$$V_{dc} = \frac{3V_{mL}}{\pi}\cos(\alpha+\mu) + \frac{3\omega L_s}{\pi} I_{dc}$$

$$= \frac{3\times565.68}{\pi}\cos(60+\mu) + \frac{3\times2\pi\times50\times2.6\times10^{-3}}{\pi}\times25$$

$$250 = 540.18\cos(60+\mu) + 19.5$$

$$\cos(60+\mu) = 0.4267$$

$$60+\mu = \cos^{-1}(0.4267)$$

$$60+\mu = 64.74.$$

$$\mu = 64.74 - 60$$

$$\boxed{\mu = 4.74^\circ}$$

29) A 3 $\phi$ Full converter charges a battery from a 3 $\phi$ supply of 230V, 50HZ. The battery EMF is 200V and its internal resistance is 0.5Ω. on account of inductance connected in series with the battery charging current is constant at 20A. compute the firing angle and the supply power factor.

Soln

Average o/p voltage $V_{dc} = E + I_{dc} R_a$

$$= 200 + 20 \times 0.5 = 210V$$

$$V_{dc} = \frac{3V_{mL}}{\pi} \cos\alpha = 210V$$

$$210 = \frac{3 \times \sqrt{2} \times 230}{\pi} \cos\alpha .$$

$$\cos\alpha = 0.676$$

$$\alpha = \cos^{-1}(0.676)$$

$$\boxed{\alpha = 47.46^{\circ}}$$

RMS value of the supply current $I_s$ over $\pi$ radian is

$$I_s = I_{dc}\sqrt{2/3}$$

$$= 20\sqrt{2/3}$$

$$= 16.33A .$$

RMS value of output current

$$I_{orms} = I_{dc} = 20A$$

power delivered to the load

$$= E I_{dc} + I_{orms}^2 R$$

$$= 200 \times 20 + (20)^2 \times 0.5$$

$$= 4200W$$

Now $\sqrt{3} V_s I_s \cos\phi = 4200W$

Input supply power factor $\cos\phi = \dfrac{4200}{\sqrt{3} \times 230 \times 16.33}$

$\cos\phi = 0.646$ lag.

30) Three phase semiconverter is operated from a three phase Y connected 220V, 60Hz, supply and the load resistance is $R = 10\Omega$. If the average output is 25% of the maximum possible average o/p voltage. calculate (i) delay angle. ii) rms and average o/p currents iii) average and rms thyristor currents iv) the rectification efficiency (v) TUF

Soln

$V_L = 220V$, $f = 60Hz$, $R = 10\Omega$

Phase voltage $V_p = \dfrac{220}{\sqrt{3}} = 127V$

peak voltage $V_m = \sqrt{2}\, V_p = \sqrt{2} \times 127$

$= 179.6V$

$V_{dc\,max} = \dfrac{3\sqrt{3}\, V_m}{\pi} = \dfrac{3\sqrt{3} \times 179.6}{\pi} = 297V$

$V_n = \dfrac{V_{dc}}{V_{dc\,max}} = \dfrac{297}{}$

$= 0.25$

$V_{dc} = V_n \times V_{dc\,max} = 0.25 \times 297 = 74.26V$

i) Delay angle :

$V_n = 0.5(1 + \cos\alpha)$

$\alpha = 120°$

ii) Average o/p current :

$$I_{dc} = \frac{V_{dc}}{R} = \frac{74.26}{10} = 7.426A.$$

$$V_{orms} = \sqrt{3}V_m\left[\frac{3}{4\pi}\left(\frac{2\pi}{3}+\sqrt{3}\cos^2\alpha\right)\right]^{1/2}$$

$$= \sqrt{3}\times 179.6\left[\frac{3}{4\pi}\left(\frac{2\pi}{3}+\sqrt{3}\cos^2 120^\circ\right)\right]^{1/2}$$

$$= 119.1V$$

RMS o/p current $I_{orms} = \frac{V_{orms}}{R} = \frac{119.1}{10} = 11.91A.$

iii) Average thyristor current :

$$I_{TA} = \frac{I_{dc}}{3} = \frac{7.426}{3} = 2.46A.$$

RMS thyristor current :

$$I_{TR} = \frac{I_{orms}}{\sqrt{3}} = \frac{11.91}{\sqrt{3}} = 6.88A.$$

iv) Rectification efficiency :

$$\eta = \frac{V_{dc}I_{dc}}{V_{orms}I_{orms}} = \frac{74.26\times 7.426}{119.1\times 11.91}\times 100$$

$$= 38.8\%.$$

v) Transformer utillization factor TUF :

$$TUF = \frac{P_{dc}}{3V_sI_s}$$

$$I_s = I_{orms}\sqrt{2/3} = 9.73A.$$

$$TUF = \frac{74.26\times 7.426}{3\times 127\times 9.73} = 0.1488$$